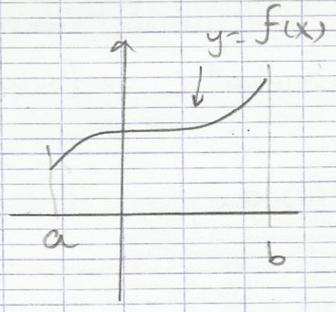
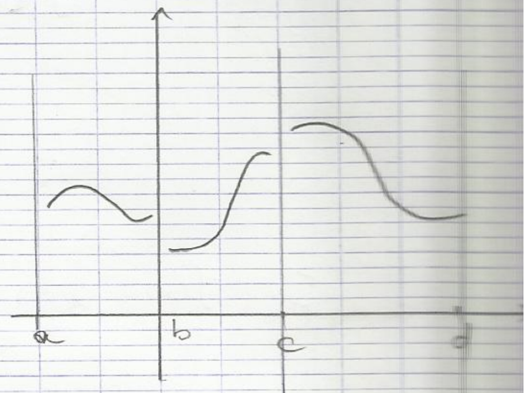
6.5 Average Value of a Function

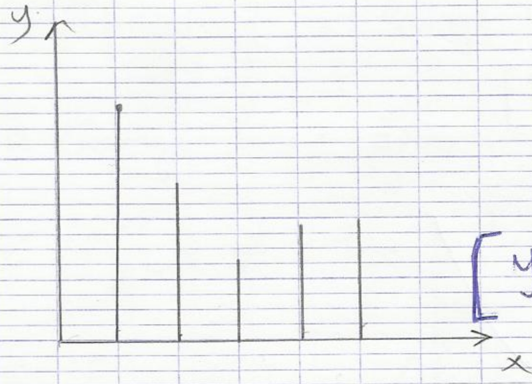
A) Continuous f^n:



Practically, in real life, we often use the Piece wise Cont f

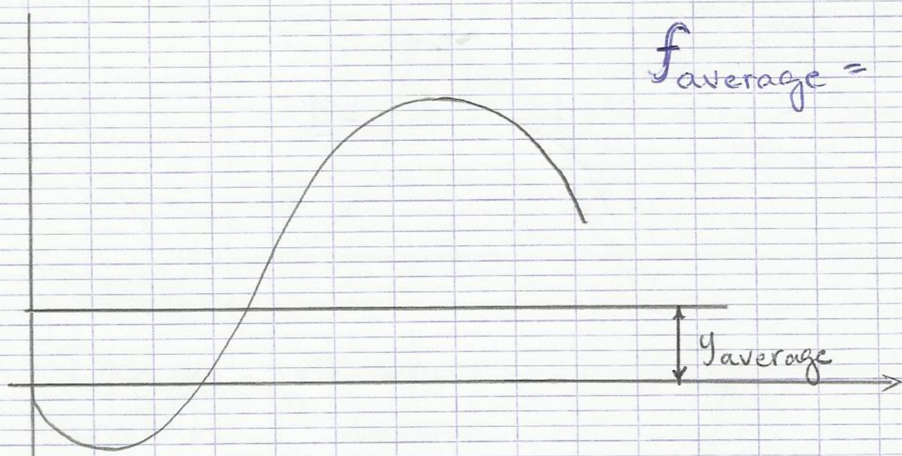


B) Discrete f^n:



$$\left[\, y_{average} = \frac{y_1 + y_2 + \cdots + y_n}{n} \,\right]$$

Continuous:



$$f_{average} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

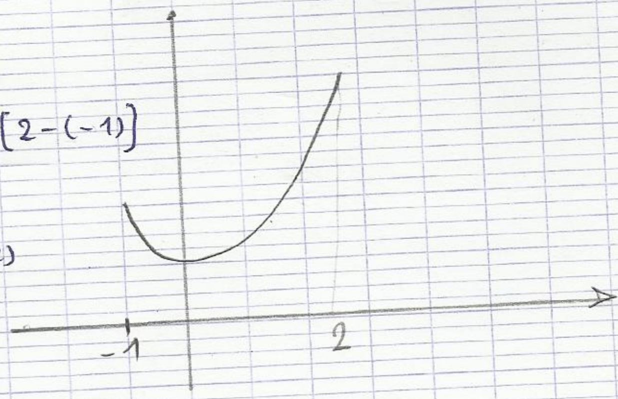
▷ Mean Value Theorem for integrals

If f is continuous f^n on the closed Interval $[a, b]$
Then there exists at least one n^b c on $[a, b]$

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\, dx$$

$$\therefore \int_a^b f(x)\, dx = f(c)\,(b-a)$$

Ex/ $f(x) = x^2 + 1$ on $[-1, 2]$



$$\int_{-1}^{2} (1+x^2)\, dx = f(c)\,[2-(-1)]$$

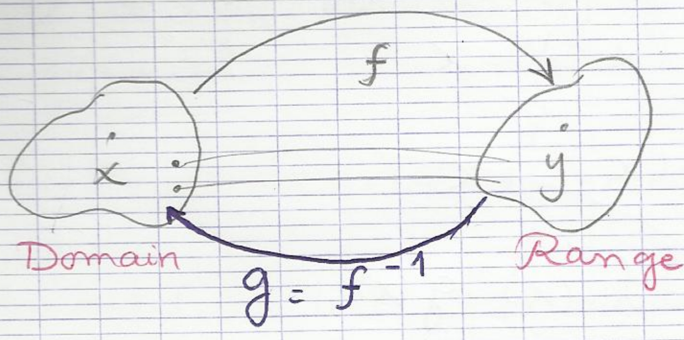
$$\left[x + \frac{x^3}{3}\right]_{-1}^{2} = 3 f(c)$$

$$f(c) = f_{av} = 2$$

$$1 + c^2 = 2 \quad \therefore c^2 = 1 \quad \therefore c = \pm 1$$

Chapter 7

7.1 Inverse functions:



1. Domain f^{-1} = Range f

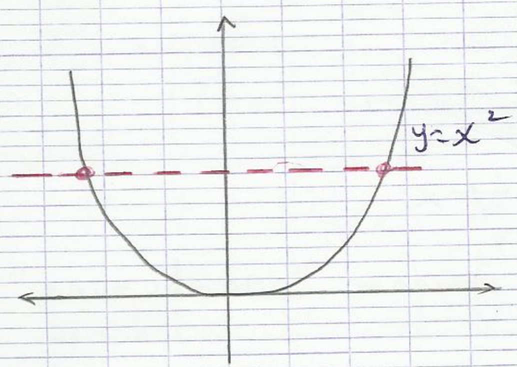
2. Range f^{-1} = Domain f

3. $f^{-1}(x) \neq \frac{1}{f(x)}$

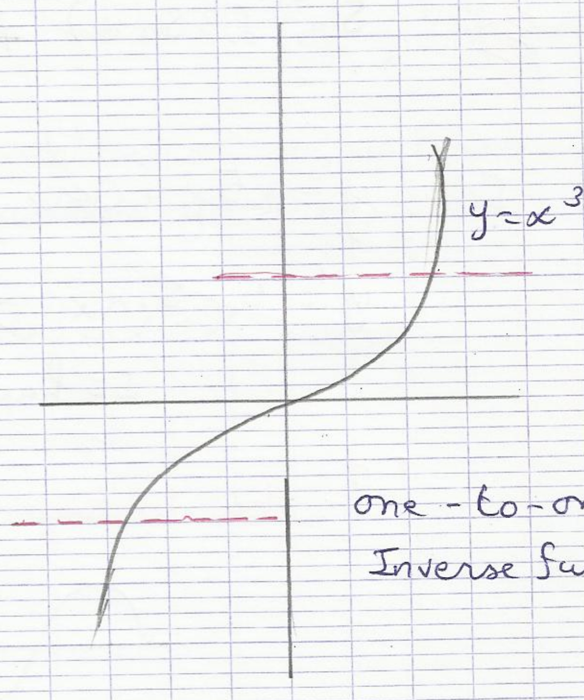
4. $f^{-1}(f(x)) = f(f^{-1}(x)) = x$

5. Every Incr / decr f^n are one-to-one

Intro



This is not an Inverse f^n because it's not 1to1 since $x = \pm 2 \rightarrow y = 4$ and the horizontal line intersects in more than one point



one-to-one Inverse function

Ex: Let $f(x) = x^3$, $g(x) = x^{\frac{1}{3}}$ .. find the relation bet. f and g

Sol. $f(g(x)) = (x^{\frac{1}{3}})^3 = x$

$g(f(x)) = (x^3)^{\frac{1}{3}} = x$

$\therefore f^{-1}(x) = g(x)$

$\therefore$ f is the inverse f^n of g and vice versa

Ex: $\frac{d}{dx} \tan^{-1}(\tan x^3) = \frac{d}{dx} x^3 = 3x^2$

Ex: $\ln e^{x^2} \rightarrow x^2$

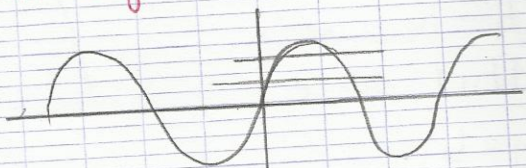
Ex: $\sin^{-1}(3 \sin x) \neq x$

Ex: $y = 5x + \cos x$

$y' = 5 - \sin x > 0 \ \forall x$

$\therefore$ It has an inverse "one-to-one"

? How do trig functions have inverse???



In fact, we restrict the domain .. this is how the calculator treats sinx as ex.:

➢ One-to-one f^n:

[1] f is called one-to-one if it never takes on the same value twice, that is

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2$$

Horizontal line test: f is one-to-one iff no horizontal line intersects more than once

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$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

➢ Find the inverse f^n of one-to-one f^n:

[1] $y = f(x)$

[2] solve the equation for x in terms of y (if possible)

[3] To express f^{-1} as a function of x, interchange x and y. The resulting equation $y = f^{-1}(x)$

Ex: Find $f^{-1}(x)$ of $f(x) = x^3 + 2$

[1] check it's one-to-one

[2] $y = x^3 + 2$

[3] $x^3 = y - 2$

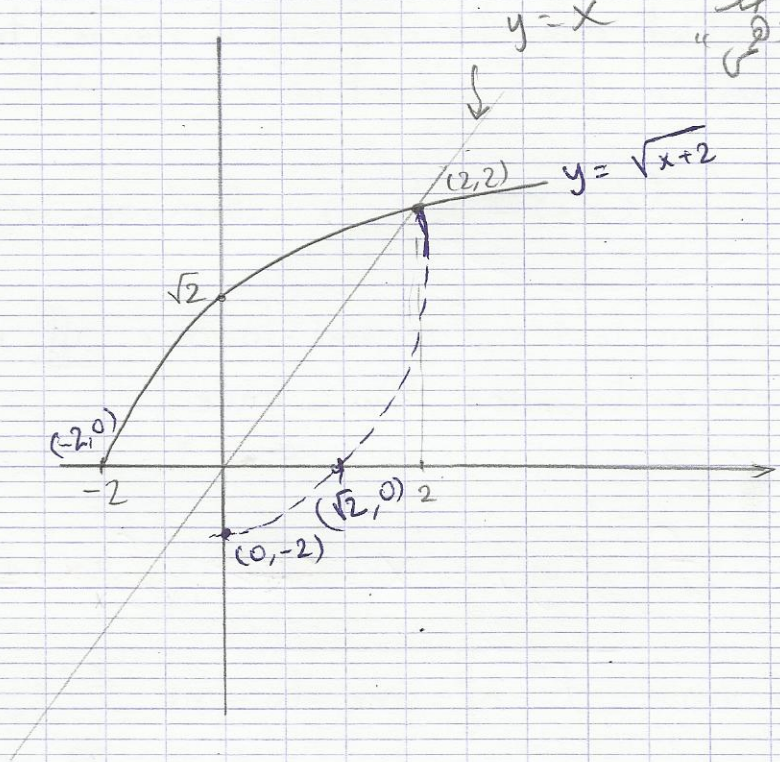
$x = (y-2)^{1/3}$

[4] $f^{-1}(x) = (x-2)^{1/3}$

[5] check → $f(f^{-1}(x)) = ((x-2)^{1/3})^3 + 2 = x$

$f^{-1}(f(x)) = (x^3 + 2 - 2)^{1/3} = x$ ✓

Ex: Find f^{-1} of $f(x) = \sqrt{x+2}$ and sketch f and f^{-1}



Ex: Sketch ~~f~~ $\ln x$ as the inverse of e^x.